A Numerical Approach for Analyzing Three-Dimensional Steady-State Rolling Contact Including Creep Using a Fast Semi-Analytical Method

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This article presents a new rolling contact solver using a semi-analytical method (SAM) to analyze three-dimensional steady-state rolling contacts, including the effects of creep. This new solver includes both the normal and tangential contact issues for the pressure and shear tractions, respectively. The accuracy and efficiency of the present method are demonstrated by comparison to existing analytical and numerical solutions. Rolling contact problems for a smooth infinite roller pressed against a half space with either a single asperity or a sinusoidal wave are investigated. The results show the complexity of pressure and shear traction. Asperities produce higher localized elastic bodies, in which a contact of smooth surfaces and Amon- ton’s law of friction were assumed. Later, Nowell and Hills (3), (4) investigated the rolling contact of two dissimilar cylinders and drew stick-slip zone maps. In their studies, a series of Cheby- shev polynomials were employed to derive the shear tractions. For three-dimensional contacts, Johnson (5) and Vermeulen and Johnson (6) obtained a simple expression for traction and creep; however, the shear traction and stick regions at the leading edge were not accurate. Haines and Ollerton (7) developed a strip theory to solve elliptical contacts, which yielded good results when the ellipse was narrow in the rolling direction. One of the most important contributions was made by Kalker (8)–(10), who proposed several theories and numerical approaches to solve two- or three-dimensional rolling contacts with creep. Within about 30 years, Kalker’s theories became widely used in rail contact analysis. Using Kalker’s numerical methods, a well-known software Contact (http://www.kalkersoftware.org/) was developed. However, this rolling contact solver involves complicated calculations that require excessive computational time.

INTRODUCTION

Rolling contacts are commonly found in engineering systems, such as rail-wheel systems, rolling bearings, and gears. When two bodies are in a rolling contact with friction and transmit tangential load to each other, the surface elastic deformations at the interface between the two contact surfaces usually do not match. There exist stick areas, where two mated points share the same speed, and a slip area, where two mated points are subjected to different velocities. Moreover, the unmatched deformations will lead to a difference in velocity between the two bodies, which is called creep. Reynolds (1) first described creep in rolling contacts, where he found that the distance through which a cylinder moved during one revolution is not equal to its circumference. This phenomenon is called creep. Carter (2) presented analytical solutions of two-dimensional rolling contact between identical elastic bodies, in which a contact of smooth surfaces and Amon- ton’s law of friction were assumed. Later, Nowell and Hills (3), (4) investigated the rolling contact of two dissimilar cylinders and drew stick-slip zone maps. In their studies, a series of Cheby- shev polynomials were employed to derive the shear tractions. For three-dimensional contacts, Johnson (5) and Vermeulen and Johnson (6) obtained a simple expression for traction and creep; however, the shear traction and stick regions at the leading edge were not accurate. Haines and Ollerton (7) developed a strip theory to solve elliptical contacts, which yielded good results when the ellipse was narrow in the rolling direction. One of the most important contributions was made by Kalker (8)–(10), who proposed several theories and numerical approaches to solve two- or three-dimensional rolling contacts with creep. Within about 30 years, Kalker’s theories became widely used in rail contact analysis. Using Kalker’s numerical methods, a well-known software Contact (http://www.kalkersoftware.org/) was developed. However, this rolling contact solver involves complicated calculations that require excessive computational time.

The finite element method (FEM) has also been used to analyze rolling contact, including such effects as crack propagation (Bogdanski, et al. (11); Liu, et al. (12)), inelastic behaviors (Bijak-Zochowski and Marek (13); Jiang, et al. (14); Xu and Jiang (15); Wen, et al. (16)), and rolling contact fatigue (Jiang and Schi- toglu (17)). In addition, boundary element methods (BEMs) have been employed to simulate 2D or 3D rolling contact (González and Abascal (18); Abascal and Rodríguez-Tembleque (19); Rodríguez-Tembleque and Abascal (20)) and wear (Rodriguez- Tembleque, et al. (21)). However, using an FEM or BEM to solve rolling contact with creep is time consuming.

Semi-analytical methods (SAMs) use analytical solutions of displacements and stresses due to unit pressure or shear tractions in the form of influence coefficients. The conjugate gradient method (CGM; Polonsky and Keer (22)) and the fast Fourier transform (FFT) technique (Liu, et al. (23)) have been adopted into the normal and sliding SAM contact solvers. SAMs have been used successfully in elasto-plastic contact (Jacq, et al. (24); Boucly, et al. (25); Chen, et al. (26); Chen and Wang (27); Wang,
As shown in Fig. 1, the grids for results are expressed by solid displacements, obtained from these fundamental solutions. The influence coefficients, which relate stress to stress or displacements, are usually in the form of a Green's function, and the final results are obtained through superposition. If the grid sizes are equal, the FFT calculations due to total pressure or shear tractions can be calculated in terms of the infinite half-space solutions of Boussinesq:

\[
\tilde{u}_{ij} = \hat{\lambda} \xi_i \xi_j
\]

where \( \hat{\lambda} \) is the stress concentration factor, \( \xi_i \) and \( \xi_j \) are the coordinates of the grid (mm), and \( \tilde{u}_{ij} \) is the surface displacement (mm). The normal displacement \( u_z \) caused by this uniform pressure or shear tractions are usually analytically derived as influence coefficients. Note that although some singular behavior of stresses will arise at the edge of a region, values are taken from the red nodes. The stresses or displacements due to total pressure or shear tractions can be calculated through superposition. If the grid sizes are equal, the FFT can be employed to calculate the convolution, which greatly reduces computational time.

The normal displacement \( u_z \) caused by a distributed normal pressure can be calculated in terms of the infinite half-space solution of Boussinesq:

\[
\tilde{u}_{ij}^P = \hat{\lambda} \xi_i \xi_j
\]
where
\[ \frac{1}{E^*} = \frac{1 - v_1^2}{E_1} + \frac{1 - v_2^2}{E_2} \]  \[2\]

For numerical computations, Eq. [1] is rewritten in discrete form:
\[ u_{zz} = \sum_{(k,l) \in A_c} C_{uz}^{p}(i-k,j-l)p_{kl} \]  \[3\]

where \( C_{uz}^{p} \) is defined as the pressure-normal displacement influence coefficient. This convolution product is calculated by using the FFT and inverse FFT,
\[ u_{zz} = C_{uz}^{p} \ast p = \text{IFFT}(\tilde{C}_{uz}^{p} \cdot \tilde{p}) \]  \[4\]

where the asterisk represents convolution in this context. The influence coefficients need to be determined in advance using Green's function and accurate interpolations to the integrand in Eq. [1]. The pressure-displacement influence coefficients were deduced by applying uniform pressure to a small rectangular area. The expression for the elastic deformation caused by the uniform pressure distributed on a rectangular area was obtained by Love (39). Based on his results, the pressure-normal displacement...
influence coefficient can be written as follows (Love (39)):

\[
\pi E^* \cdot C_{ij}^m = y_m \ln \left( \frac{x_m + \sqrt{x_m^2 + y_m^2}}{x_m} \right) + x_m \ln \left( \frac{y_m + \sqrt{x_m^2 + y_m^2}}{y_m} \right) - y_m \ln \left( \frac{x_p + \sqrt{x_p^2 + y_p^2}}{x_p} \right) - x_p \ln \left( \frac{y_p + \sqrt{x_p^2 + y_p^2}}{y_p} \right) - y_p \ln \left( \frac{x_m + \sqrt{x_m^2 + y_m^2}}{x_m} \right) - x_m \ln \left( \frac{y_p + \sqrt{x_p^2 + y_p^2}}{y_p} \right) + y_p \ln \left( \frac{x_p + \sqrt{x_p^2 + y_p^2}}{x_p} \right) + x_p \ln \left( \frac{y_m + \sqrt{x_m^2 + y_m^2}}{y_m} \right)
\]

where \(x_m = x_j + \Delta x / 2, x_p = x_j - \Delta x / 2, y_m = y_j + \Delta y / 2, y_p = y_j - \Delta y / 2, \) and \(\Delta x \) and \(\Delta y\) denote grid sizes along the \(x\) and \(y\) axes, respectively.

Moreover, the surface displacements due to pressure \(P\) and shear tractions \(q_x\) and \(q_y\) can be expressed by,

\[
\begin{bmatrix}
u_x \\
u_y \\
u_z
\end{bmatrix} = \begin{bmatrix}
u_{xx} + \nu_{xy} + \nu_{xz} \\
u_{yx} + \nu_{yy} + \nu_{yz} \\
u_{zx} + \nu_{zy} + \nu_{zz}
\end{bmatrix} = \begin{bmatrix}C_{xx} & C_{xy} & C_{xz} \\
C_{yx} & C_{yy} & C_{yz} \\
C_{zx} & C_{zy} & C_{zz}
\end{bmatrix} \begin{bmatrix}q_x \\
q_y \\
P
\end{bmatrix}
\]

where \(\nu_x = \nu_{xx} + \nu_{xy} + \nu_{xz}\), \(\nu_y = \nu_{yx} + \nu_{yy} + \nu_{yz}\), and \(\nu_z = \nu_{zx} + \nu_{zy} + \nu_{zz}\) are surface displacements parallel to the \(x\), \(y\), and \(z\) directions. \(C_{xx}, C_{xy}, C_{xz}\), \(C_{yx}, C_{yy}, C_{yz}\), \(C_{zx}, C_{zy}, C_{zz}\) are related influence coefficient matrices. The expressions of the influence coefficients were derived by Love (39) and James and Busby (40). The explicit expressions of predetermined influence coefficients can be found in Chen and Wang (33). Likewise, the stress due to pressure and shear tractions can be calculated by means of the related influence coefficients, which can be found in the literature (Kalker (9); Ahmadi, et al. (41); Hills, et al. (42); Liu and Wang (43)). SAMs provide considerable savings in the computational effort due to the FFT method and the fact that only the contact area of interest needs to be meshed.

**ROLLING CONTACT MODEL**

The rolling contact between two bodies can be defined with respect to a Cartesian system, as shown in Fig. 2 where \(O\) is the origin. The \(z\) axis is normal to the contact surface and the \(x-y\)
plane is the tangential plane to the contact surface at \( O \). The Cartesian components of linear velocity \( V \), \( U \) and angular velocities \( \omega_1 \), \( \omega_2 \), and \( \omega_3 \) are considered. In the numerical process, the contact solution procedure is divided into two steps: the normal contact is determined first and then the tangential contact is solved. It is worth mentioning that for the contact of dissimilar materials, the normal and tangential contact solvers need to be coupled due to the interaction terms arising from the pressure and shear tractions.

**Normal Effect of Contact**

The contact problem method of solution requires determining the minimum of the total complementary potential energy, which is transformed into an equivalent problem that obeys the Kuhn-Tucker complementary conditions:

\[
p \geq 0, \quad g \geq 0, \quad \text{and} \quad p \cdot g = 0 \quad [7]
\]

where \( g \) denotes the surface normal gap and can be expressed by:

\[
g = h_0 - \delta_z + u_z \quad [8]
\]

In the above equation, \( h_0 \) is initial body separation between two surfaces, \( \delta_z \) is the \( z \)-direction rigid displacement of the two surfaces, \( u_z = u_{z1} + u_{z2} \) is the normal displacement, and \( u_{z1} \) and \( u_{z2} \) denote the surface normal displacements due to shear tractions \( q_s \) and \( q_t \) and pressure \( p \), respectively. The displacements can be calculated in terms of influence coefficients by using the FFT method, and thus Eq. [8] can be rewritten as

\[
g = h_0 - \delta_z + u_{z1} + u_{z2} + C_p^j \cdot p \quad [9]
\]

The normal load \( W \) balance equation is

\[
\int_{\lambda} \int \rho(x,y)dx
dy = W \quad [10]
\]

Once the displacements \( u_{z1}, u_{z2} \) are obtained, Eq. [9] becomes a set of linear equations for the unknown pressure. The influence coefficients matrix \( C_p^j \) is a real symmetric matrix, so that the CGM can be employed to search for solutions to the linear equations. The constraint Eqs. [7] and [10] are enforced in the iteration steps. In the contact of identical materials, deformations \( u_{z1} \) and \( u_{z2} \) are zero, and thus the pressure profile is unaltered by the shearing tractions.

**Tangential Effect of Contact**

The shear tractions can be found through tangential contact equations. The governing equations in steady-state rolling can be expressed by (Johnson (44))

\[
\dot{s}_x/V = \xi_x - \psi y - \partial u_x/\partial x
\]

\[
\dot{s}_y/V = \xi_y + \psi x - \partial u_y/\partial x
\]

where \( V \) is the rolling velocity along the \( x \) direction; \( s_x \) and \( s_y \) denote the slip between the two surfaces parallel to the \( x \) and \( y \) directions; \( \xi_x, \xi_y, \) and \( \phi \) are the longitudinal, lateral, and spin creepages, respectively. Note that creepage is defined as the difference in velocities between the contacting surfaces divided by the mean velocity. The contact area is divided into zones of stick and slip, where the shear tractions in the stick and slip regions are assumed to obey the following conditions:

In stick regions: \( \sqrt{q_x^2 + q_y^2} \leq \mu \rho_p \) and \( \sqrt{\dot{s}_x^2 + \dot{s}_y^2} \neq 0 \) [12]

In stick regions: \( \sqrt{q_x^2 + q_y^2} = \mu \rho_p \) and \( \sqrt{\dot{s}_x^2 + \dot{s}_y^2} \neq 0 \) [13]
The directions of shear tractions must oppose the slip velocity; that is,

\[ q[x] = -\hat{s}[q] \]  \hspace{1cm} (14)

Furthermore, in stick regions, \( \hat{s}_x = s_y = 0 \), and we have

\[ \frac{\partial u_x}{\partial x} = \xi_x - \varphi y \]
\[ \frac{\partial u_y}{\partial x} = \xi_y + \varphi x \]  \hspace{1cm} (15)

Integrating with respect to \( x \), gives the result:

\[ u_x = \xi_x x - \varphi xy + f(y) \]
\[ u_y = \xi_y x + \varphi_2/2 + g(y) \]  \hspace{1cm} (16)

where \( f(y) \) and \( g(y) \) are functions of \( y \), to be determined from the iteration. Spin is not considered in the present study. The mating surfaces will be meshed, and only the values on the grids are considered. Discretizing Eq. \( (16) \) results in the following:

\[ u_{xij} = \xi_x x_i + f_j \]
\[ u_{yij} = \xi_y x_i + g_j \]  \hspace{1cm} (17)

The displacements can be calculated in terms of influence coefficients using the FFT method, and Eq. \( (17) \) then becomes

\[
\begin{bmatrix}
C_{u_x} \quad C_{u_y} \quad C_{u_z} \\
C_{u_x} \quad C_{u_y} \quad C_{u_z} \\
C_{u_x} \quad C_{u_y} \quad C_{u_z}
\end{bmatrix}
\begin{bmatrix}
q_{xij} \\
q_{yij} \\
p_{ij}
\end{bmatrix}
= 
\begin{bmatrix}
\xi_x x_i + f_j \\
\xi_y x_i + g_j
\end{bmatrix}
\]  \hspace{1cm} (18)

Pressure \( p \) can be determined through normal contact as described in the previous subsection. Thus, the unknown variables are shear tractions \( q_x \) and \( q_y \). Equation \( (18) \) is a set of linear equations; the influence coefficients form a symmetric matrix. The CGM can be employed to solve Eq. \( (18) \) with constraint conditions, Eqs. \( (12) \)–\( (14) \). Details of the iteration procedure are summarized as follows:

1. Set initial values for creep ratios \( \xi_{xij}^{(0)} \) and \( \xi_{yij}^{(0)} \), shear tractions \( q_x^{(0)} = 0, q_y^{(0)} = 0 \), initial conjugate directions \( t_x^{(0)} = 0, t_y^{(0)} = 0 \), and initial residual variable \( r^{(0)} = 1 \). The pressure distribution is obtained from the normal contact solution first.
2. Determine the stick region. If \( \sqrt{q_{xij}^{(0)} + q_{yij}^{(0)}} < \mu p_{ij} \), point \( (i, j) \) moves into the stick region. Then, calculate \( f_j \) and \( g_j \):

\[
\begin{bmatrix}
f_j \\
g_j
\end{bmatrix}
= 
\begin{bmatrix}
C_{u_x} \quad C_{u_y} \\
C_{u_x} \quad C_{u_y} \\
C_{u_x} \quad C_{u_y}
\end{bmatrix}
\begin{bmatrix}
q_{xij} \\
q_{yij}
\end{bmatrix}
- 
\begin{bmatrix}
\xi_x x_i \\
\xi_y x_i
\end{bmatrix}
\]  \hspace{1cm} (19)

Note that \( f_j \) and \( g_j \) are only functions of \( j \).
3. Let \( r_x \) and \( r_y \) be the residual vectors at the \( n \)th step:

\[
\begin{bmatrix}
r_x^{(n)} \\
r_y^{(n)}
\end{bmatrix}
= 
\begin{bmatrix}
C_{u_x} \quad C_{u_y} \\
C_{u_x} \quad C_{u_y} \\
C_{u_x} \quad C_{u_y}
\end{bmatrix}
\begin{bmatrix}
q_{xij}^{(n)} \\
q_{yij}^{(n)}
\end{bmatrix}
- 
\begin{bmatrix}
\xi_x x_i + f_j \\
\xi_y x_i + g_j
\end{bmatrix}
\]  \hspace{1cm} (20)

4. Calculate the sum of squares of the residual vectors in the stick regions:

\[ r^{(n)} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left( r_x^{(n)} ight)^2 + \left( r_y^{(n)} ight)^2 \quad \text{for} \ (i, j) \in A_{st} \]  \hspace{1cm} (21)

Here \( A_{st} \) denotes the stick regions.
5. Calculate conjugate directions \( t_x^{(n)} \) and \( t_y^{(n)} \) in all stick and slip regions. If \( \frac{r^{(n)}}{r^{(n-1)}} < 1 \), renew the conjugate directions as follows:

\[
\begin{bmatrix}
t_x^{(n)} \\
t_y^{(n)}
\end{bmatrix}
= 
\begin{bmatrix}
t_x^{(n-1)} \\
t_y^{(n-1)}
\end{bmatrix}
+ \frac{r^{(n)}}{r^{(n-1)}} \cdot 
\begin{bmatrix}
t_x^{(n-1)} \\
t_y^{(n-1)}
\end{bmatrix}
\]  \hspace{1cm} (22)

Otherwise, proceed to the next step.

---

Fig. 8—Dimensionless pressure \( p_x/p_x \) and shear traction \( q_x/p_x \) along the x-direction during the rolling process. The center of the roller was located at (a) \( L = a \); (b) \( L = 0.5a \); (c) \( L = 0 \); (d) \( L = -0.5a \); and (e) \( L = -a \). (color figure available online.)
6. Update the step length, \( \tau^{(n)} \):

\[
\tau^{(n)} = \frac{\left[ C_{x}^{(0)} x_{x}^{(0)} \right] \cdot \left[ C_{y}^{(0)} y_{y}^{(0)} \right]^{T}}{\left[ C_{x}^{(0)} x_{y}^{(0)} \right] \cdot \left[ C_{y}^{(0)} y_{y}^{(0)} \right]}
\]

[23]

The convolution in the above equation can also be calculated using the FFT algorithm.

7. Modify the shear traction distribution by:

\[
\begin{bmatrix}
  q_{x}^{(n+1)} \\
  q_{y}^{(n+1)}
\end{bmatrix} = \begin{bmatrix}
  q_{x}^{(0)} \\
  q_{y}^{(0)}
\end{bmatrix} - \tau^{(n)} \cdot \begin{bmatrix}
  \xi_{x}^{(0)} \\
  \xi_{y}^{(0)}
\end{bmatrix}
\]

[24]

If \( \sqrt{q_{xij}^2 + q_{yij}^2} > \mu_f p_{ij} \), node \((i, j)\) is set to be in the slip regions, and \( q_{x} \) and \( q_{y} \) are modified by:

\[
\begin{bmatrix}
  q_{x(i,j)} \\
  q_{y(i,j)}
\end{bmatrix} = \frac{\mu_f p_{ij}}{\sqrt{q_{xij}^2 + q_{yij}^2}} \begin{bmatrix}
  q_{xij} \\
  q_{yij}
\end{bmatrix}
\]

[25]

If \( \Delta_{x,\Delta_y} \sum_{i=1}^{M} \sum_{j=1}^{N} (|q_{x(i,j)}^{(n+1)} - q_{x(i,j)}^{(0)}| + |q_{y(i,j)}^{(n+1)} - q_{y(i,j)}^{(0)}|) < \text{eps} \), go to step 8; otherwise, go to step 2. Here, \( \text{eps} \) is a predetermined error control.

8. Compute the tangential forces, \( F_{x}' \) and \( F_{y}' \):

\[
F_{x}' = \Delta_{x,\Delta_y} \sum_{i=1}^{M} \sum_{j=1}^{N} q_{x(i,j)} \quad F_{y}' = \Delta_{x,\Delta_y} \sum_{i=1}^{M} \sum_{j=1}^{N} q_{y(i,j)}
\]

[26]

If \( |F_{x} - F_{x}'| + |F_{y} - F_{y}'| < \text{error} \), the iteration stops.

9. Update creep ratios along the \( x \) and \( y \) directions as follows and go to step 1:

\[
\xi_{x} = \xi_{x} F_{x}/F_{x}' \quad \xi_{y} = \xi_{y} F_{y}/F_{y}'
\]

[27]

The initial creep ratios, \( \xi_{x}^{(0)} \) and \( \xi_{y}^{(0)} \), can be given according to the following equations (Johnson (44)):

\[
\begin{cases}
\xi_{x}^{(0)} = -\frac{\mu_f a}{R} \left( 1 - (1 - F_{x}/\mu_f W)^{1/2} \right) \\
\xi_{y}^{(0)} = -\left( \frac{2(1 + v_{1})}{E_{1}} + \frac{2(1 + v_{2})}{E_{2}} \right) \mu_f p_{h} \quad \text{for line contact} \\
&\times (1 - (1 - F_{y}/\mu_f W)^{1/2})
\end{cases}
\]

[28]

or

\[
\begin{cases}
\xi_{x}^{(0)} = -\frac{3\mu_f W}{16a^2} \left( \frac{4 - 3v_{1}}{E_{1}} (1 + v_{1}) \right) \\
&+ \left( \frac{4 - 3v_{2}}{E_{2}} (1 + v_{2}) \right) (1 - (F_{x}/\mu_f W)^{1/3}) \\
&\quad \text{for point contact} \\
\xi_{y}^{(0)} = -\frac{3\mu_f W}{16a^2} \left( \frac{4 - v_{1}}{E_{1}} (1 + v_{1}) \right) \\
&+ \left( \frac{4 - v_{2}}{E_{2}} (1 + v_{2}) \right) (1 - (F_{y}/\mu_f W)^{1/3})
\end{cases}
\]

[29]

where \( \frac{1}{R} = \frac{1}{R_{x}} + \frac{1}{R_{y}} \), \( a \) is the semi-contact width for a line contact or the Hertzian contact radius for a point contact, and \( p_{h} \) is the maximum Hertzian pressure.

VERIFICATION AND VALIDATION OF THE MODEL

Rolling Contact between an Infinite Roller and a Half Space

The line rolling contact shown in Fig. 3a is considered in this section. An infinite roller is pressed against a half space with rolling velocity \( V \) and transmits a tractive force or a tangential force. The roller radius, \( R \), is 20 mm. The roller and half space have the same elastic properties; that is, Young’s modulus \( E = \)

![Fig. 9 — Dimensionless pressure \( p/p_{h} \) and shear traction \( q_{x}/p_{h} \) for different amplitudes \( A \) and wavelengths \( B \): (a) \( A = 1 \times 10^{-3}, B = 1 \); (b) \( A = 2 \times 10^{-3}, B = 1 \); and (c) \( A = 1 \times 10^{-3}, B = 2 \). (color figure available online.)](color figure available online.)
Three-Dimensional Rolling Contact

210 GPa and Poisson’s ratio $\nu = 0.3$. The normal load $W$ per unit length is fixed to 100 N/mm. The friction coefficient, $\mu_f$, is chosen as 0.1. The results are normalized by the maximum Hertzian pressure, $p_h = 428.5327$ MPa, and the semi-contact width, $a = 0.1486$ mm.

In the numerical analysis, the line-contact results were achieved by using the three-dimensional contact solver mentioned in the previous section. The key is that before performing the FFT, the pressure and shear tractions require duplicated padding periodically in the length direction—that is, the $y$
direction-and zero padding in the nonperiodic direction—that is, the \( x \) direction. The extended pressure or shear tractions are illustrated in Fig. 3b. More details were presented in Chen, et al. (26). This treatment allows the inclusion surface asperities of any geometry and orientation.

Figure 4a shows the distributions of shear tractions \( q_x \) along the rolling direction \( x \) for different tractive forces and Fig. 4b plots the creep curve. The analytical solutions were achieved according to Carter’s paper (Carter (2)). Good agreement between the present results and the analytical solution validate the proposed method.

**Rolling Contact of Two Spheres**

In this section, two identical spheres are pressed together and roll over each other, as shown in Fig. 5a. The radius \( R = 337.5 \). The shear modulus \( G = 1 \) and the Poisson’s ratio \( \nu = 0.28 \). The friction coefficient is \( \mu_f = 0.4013 \). The normal load \( W \) is \( \left( \frac{7}{9} \right)^3 \) and tangential force \( F_x \) is \( 0.657\mu_fW \). These parameters are the same as those in Kalker’s paper (Kalker (10), sec. 5.2.2.5). The results are normalized by the maximum Hertzian pressure, \( p_h = 0.01834 \), and the contact radius, \( a = 3.5 \). Contact ver. 11.1 software was employed to obtain Kalker’s solution as a comparison.

Figure 5b plots the shear tractions along three lines at \( y = 0.0 \), \( y = 0.5a \), and \( y = 0.75a \), parallel to the rolling direction. Moreover, Fig. 5c shows the contour plots of the dimensionless shear traction \( q_x \) using the present method; Fig. 5d is Kalker’s solution (Kalker (10)). Good agreement is seen between the present article and Kalker’s results. A computer with 2.5 GHz and i5 CPU was used for both, and the CPU times of the present method and Kalker’s are shown in Fig. 6 for different grid refinement levels. The CGM and FFT techniques also contribute to the efficiency of the current method.

**ROLLING CONTACT SURFACE WITH IRREGULARITIES**

The pressure and shear tractions will be influenced by surface roughness, which tends to induce high localized pressure and shear tractions as well as subsurface stresses. In this section, the problems of a smooth infinite roller pressed against a half space with a single asperity and a sinusoidal wave are investigated. The roller rolls with a constant velocity, and the two contacting bodies have the same elastic properties. The material properties and loading conditions are the same as those mentioned in the previous section. The size of the simulation domain is \( 6a \times 6a \) mm, which is divided into 256 \( \times \) 256 grid points for the single asperity and 512 \( \times \) 512 grid points for the sinusoidal wave. Because the deformations represent the sums of the influence of each local

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Fig. 12—Distributions of stick and slip regions during the rolling process. The center of the roller was located at (a) \( L = 0.5a \); (b) \( L = 0.25a \); (c) \( L = 0 \); (d) \( L = -0.25a \); and (e) \( L = -0.5a \). (color figure available online.)
pressure or shear traction, the interactions between the asperities are automatically taken into account.

**A Single Asperity**

A 3D asperity is assumed to be at the center of contact and the roller rolls over the asperity. The geometric shape of the asperity is described by

\[ Z(X, Y) = \bar{A} \exp(-1000(X/\bar{B})^2) \cos(2\pi X/\bar{B}) \]  \[ \text{[30]} \]

where \( X, Y, \) and \( Z \) are dimensionless coordinates normalized with \( \bar{A} \). \( \bar{A} \) is the dimensionless amplitude and \( \bar{B} \) is the dimensionless wavelength. The dimensionless pressure \( p/p_h \) and shear traction \( q/q_h \) for different amplitudes \( \bar{A} \) and wavelengths \( \bar{B} \) are plotted in Fig. 7. A larger asperity amplitude (see Fig. 7b) produces a higher pressure and shear traction than a smaller amplitude does (see Figs. 7a and 7c). Figure 8 shows the dimensionless pressure \( p/p_h \) and shear tractions \( q/q_h \) during the rolling process with fixed \( \bar{A} = 5 \times 10^{-4}, \bar{B} = 1 \), and tangential force \( F_x = 0.5\mu W \). As shown in Fig. 8a, the stick region is initially at the leading edge, which is the same as in the corresponding smooth contact. However, the pressure is higher at the leading edge due to the asperity contact; thus, a higher shear traction is needed to produce slip in this region. When the roller passes over the asperity, as shown in Fig. 8d, slip occurs at the right side of the asperity, which means that partial regions of the asperity slip. When the asperity moves to the trailing edge of the contact, the whole asperity slips.

**Sinusoidal Wave Contact**

The effects of rough surfaces are considered, in which a small patch of sinusoidal waviness is superposed on the lower half space. The waviness is characterized by its dimensionless amplitude \( \bar{A} \) and wavelength \( \bar{B} \), and the geometric shape of the sinusoidal wave is defined by

\[ Z(X, Y) = -\bar{A} \cos(2\pi X/\bar{B}) \cdot \cos(2\pi Y/\bar{B}) \]  \[ \text{[31]} \]

where \( X, Y, \) and \( Z \) are dimensionless coordinates normalized with \( \bar{A} \). The dimensionless pressure \( p/p_h \) and shear traction \( q/q_h \) for different amplitudes \( \bar{A} \) and wavelengths \( \bar{B} \) are plotted in Fig. 9. Larger \( \bar{A} \) and smaller \( \bar{B} \) produce higher pressure and shear tractions, and the contact regions are separated by the asperities (see Fig. 9a). Figure 10 shows the dimensionless pressures \( p/p_h \) and shear tractions \( q/q_h \) at different positions of the rolling contact with fixed \( \bar{A} = 0.001, \bar{B} = 1 \), and tangential force \( F_x = 0.5\mu W \). In addition, the contour plots of dimensionless shear tractions are shown in Fig. 11. As shown in Figs. 10 and 11, the profiles of shear tractions are disturbed by the asperities and the stick and slip regions are scattered. The profile of shear tractions on a local asperity is similar to the results of smooth contact. The stick regions largely occur at the leading edge, whereas the slip regions mostly emerge at the trailing edge. However, the stick regions can also be observed at the trailing edge of the contact due to the roughness effect. Furthermore, the stick and slip regions at different contact positions are presented in Fig. 12. We observe that even for the single asperity case the contact regions can be divided into stick and slip regions. In addition, the asperities located at the leading edge are more prone to stick than those at the trailing edge.

Figure 13 plots the stick ratio—that is, \( \bar{A}_s/\bar{A}_c \)—which is the stick area divided by the contact area, when the center of the roller is located at \( L = 0 \). Figures 13a and 13b show the stick ratio as a function of amplitude \( \bar{A} \) and wavelength \( \bar{B} \), respectively. As revealed in Fig. 13a, the stick ratios decrease as the asperity amplitudes increase; however, the curves of stick ratios become flat when \( \bar{A} \) is larger than 0.001. For example, in the case of \( F_x = 0.9\mu W \), the stick ratio is 0.316 for \( \bar{A} = 0 \), the smooth contact, whereas it decreases to 0.153 for \( \bar{A} = 0.001 \). Figure 13b shows

\[ F_x/\mu W = 0.3, 0.5, 0.7, 0.9 \]

\[ \bar{A} = 0.001 \]

\[ \bar{B} = 1 \]

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the influence of $B$ on stick ratio for fixed $A = 0.001$. Larger $B$ causes a higher stick ratio when $B$ is less than 2. Likewise, the curves of stick ratio become flat when $B > 2$. Furthermore, Fig. 13c shows the stick ratio in different contact areas as a function of tangential force with fixed $A = 0.001$ and $B = 1$. In this case, the contact region is divided into two parts: the half contact region in the leading direction, $S_1$, and the half contact region in the trailing direction, $S_2$. For a smooth surface contact, the stick ratio in $S_1$ is a constant; that is, the $S_1$ region is in complete stick when the tangential force $F_t/\mu W$ is less than 0.77. However, for a rough surface contact, the stick ratio in $S_1$ is not a constant but decreases with the increased tangential force. Under a constant tangential force, the contact ratio in $S_1$, $S_2$, or $S_1 + S_2$ for a rough surface contact is lower than that of a smooth surface contact. Overall, the surface asperities cause the surface stick-slip phenomenon to be more complex than in a smooth contact.

CONCLUSIONS
A numerical method for solving steady-state rolling contact based on SAM was presented. The method was validated by good agreement with the results of previous analytical solutions and with Kalker’s solutions. Moreover, the use of CGM and FFT techniques further enhances the efficiency of the new method.

This method was applied to analyze two rolling cases involving surface irregularities; that is, a single asperity and sinusoidal wavy asperities. The results showed that the stick and slip regions are influenced by the asperities, which produce higher local pressures and require a large local shear traction to produce slip in such regions. For the contact involving a surface with sinusoidal wavy asperities, the stick and slip regions are scattered. Even for a single asperity, the contact region can be divided into stick and slip regions. Moreover, the stick ratios are influenced by the amplitudes and wavelengths.

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