Name: ____________________________________________

Please also put your name on the back of the last page!

My LECTURE section starts at: 8:00am / 9:30am / 2:00pm / 3:30pm

My DISCUSSION section starts at: 9:00am / 10:00am / 11:00am / 12:00pm

On my honor I have neither given nor received help on this quiz and I will not discuss content of the quiz with other students until 5pm. (1pt) ______________________________

Sign your name above

You may NOT use notes or books or calculators or phones for this quiz. Do not ask for clarification of the questions. There are 5 problems. There is only one correct answer for all parts of Problems I through III. Answer Problem IV in the quiz booklet and show all work. Good luck!

Useful formulas

Ohm's Law: $V = iR$
Capacitor Law: $V = (1/C)q$
Energy in a capacitor: $E = \frac{1}{2} CV^2$
Inductor Law: $V = L \frac{di}{dt}$
Power: $P = iV$

Euler formula: $e^{iy} = \cos y + jsin y$; $e^{-iy} = \cos y - jsin y$

Quadratic formula for $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
### Problem I. (24 pts, 1 pts each, except where noted)

<p>| | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>The net charge on a capacitor is always approximately zero.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A resistor can store energy.</td>
<td></td>
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<tr>
<td>3</td>
<td>Inductors store energy in a magnetic field.</td>
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<tr>
<td>4</td>
<td>A capacitor is analogous to a spring.</td>
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<tr>
<td>5</td>
<td>The net charge in an electric circuit is always approximately zero.</td>
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<tr>
<td>6</td>
<td>The effective spring constant of 2 springs, ( k_1 ) and ( k_2 ), in series is ( k_{eff} = k_1 + k_2 ).</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>The effective capacitance of 2 capacitors, ( C_1 ) and ( C_2 ), in parallel is ( C_{eff} = C_1 + C_2 ).</td>
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<tr>
<td>8</td>
<td>A circuit with a resistor and capacitor in series has a time constant of ( RC ).</td>
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<tr>
<td>9</td>
<td>Current can change instantaneously in a capacitor.</td>
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<tr>
<td>10</td>
<td>Current can change instantaneously in an inductor.</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>In steady state, a capacitor seeks zero current and constant voltage.</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>In steady state, an inductor seeks zero current and zero voltage.</td>
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For questions 13-16: the following questions refer to analogous relationships between electrical and mechanical systems.

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<table>
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<tr>
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<tbody>
<tr>
<td>13</td>
<td>A resistor in an electrical circuit is analogous to a spring in a mechanical system</td>
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<tr>
<td>14</td>
<td>A constant external force in a mechanical system is analogous to a constant voltage battery in an electrical system.</td>
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<tr>
<td>15</td>
<td>A spring and damper in parallel is analogous to a capacitor and resistor in parallel.</td>
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<tr>
<td>16</td>
<td>A capacitor and inductor in parallel is analogous to a spring and mass in series.</td>
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</tbody>
</table>
Questions 17 and 19 refer to the circuit to the right.

17. Which of the following contain(s) correct KCL relationships for the circuit to the right? Choose the best option.
   A. \( i_{R2} = i_{C1} + i_{R3} \)
   B. \( i_{R3} = i_{R2} + i_{C4} \)
   C. \( i_{R3} = i_{C1} \)
   D. all of the above
   E. none of the above
   Your answer ________

18. Which of the following contain(s) correct KVL relationships for the circuit to the right? Choose the best option.
   A. \( V_{BAT0} = V_{C1} + V_{R2} + V_{C4} \)
   B. \( V_{C1} = V_{R3} \)
   C. \( V_{BAT0} = V_{R2} + V_{C4} + V_{R3} \)
   D. all of the above
   E. none of the above
   Your answer ________

19. The electrical system to the right is analogous to which of the mechanical systems below (next page)? (2 pts)

A

B

C

D

Your answer ________
Questions 20 and 22 refer to the system to the right.

20. Which of the following contain(s) correct force balance relationships for the mechanical system to the right?

A. \( f_3 = -f_1 - f_2 + f_4 \)
B. \( m_4a_4 = -f_1 \)
C. \( f_1 = f_3 - f_2 \)
D. all of the above
E. none of the above

Your answer ________

21. Which of the following contain(s) correct geometry continuity relationships for the mechanical system to the right? Use “positive to the right” sign convention for the mass velocity.

A. \( v_3 = V_4 - v_1 \)
B. \( v_1 = v_2 \)
C. \( v_1 + v_3 = V_4 \)
D. all of the above
E. none of the above

Your answer ________

22. The mechanical system above is equivalent to which of the electrical systems below? (2 pts)

A. 
B. 
C. 
D. 

Your answer ________
Problem II. (24 pts, 2 pt each)

Questions 23 – 28 refer to the circuit to the right as shown with two resistors (R3 = R5 = 100 Ω), 1 capacitor (C2 = 4F), and 1 battery (6V). C2 is initially uncharged before the switch is closed at t = 0.

23. After SW1 is first closed (t = 0+), V_{C2} is
   A. 0 v          B. 3 v          C. 6 v
   D. 12 v         E. cannot be determined
   Your answer ________

24. After SW1 is first closed (t = 0+), i_{C2} is
   A. 0 amp       B. 0.04 amp     C. 0.06 amp
   D. 1.5 amp     E. cannot be determined
   Your answer ________

25. At steady state, the voltage across C2 is
   A. 0 v          B. 3 v          C. 6 v
   D. 12 v         E. cannot be determined
   Your answer ________

26. At steady state, the current across C2 is
   A. 0 amp       B. 0.04 amp     C. 0.06 amp
   D. 1.5 amp     E. cannot be determined
   Your answer ________

27. What is the qualitative form of the current in C2 in the circuit after SW1 is closed?
   [Graph (A), Graph (B), Graph (C), Graph (D), Graph (E)]
   Your answer ________

28. What is the qualitative form of the voltage in C2 in the circuit after SW1 is closed?
   [Graph (A), Graph (B), Graph (C), Graph (D), Graph (E)]
   Your answer ________
Questions 29 - 34 refer to a different modification of the circuit from above. Consider the circuit to the right. C4 is initially not charged. Parameters are given as $R_2 = 200 \, \Omega$, $R_3 = 100 \, \Omega$, $C_4 = 1 \, \text{F}$, $L_5 = 3 \, \text{H}$, and $\text{BAT1} = 6\, \text{V}$. Then SW1 is closed at $t = 0$.

29. Before SW1 is closed, the energy stored on C4 is:
   A. 0 J   B. 9 J   C. 18 J
   D. 36 J   E. 72 J
   Your answer ________

30. After SW1 is first closed ($t = 0^+$), $V_{C4}$ is
   A. 0 v   B. 2 v   C. 4 v
   D. 8 v   E. cannot be determined
   Your answer ________

31. After SW1 is first closed ($t = 0^+$), $i_{L5}$ is
   A. 0 amp   B. 0.015 amp   C. 0.06 amp
   D. 1.5 amp   E. cannot be determined
   Your answer ________

32. At steady state, the voltage across C4 is
   A. 0 v   B. 1 v   C. 3 v
   D. 6v   E. cannot be determined
   Your answer ________

33. At steady state, the voltage across L5 is
   A. 0 v   B. 1 v   C. 3 v
   D. 6v   E. cannot be determined
   Your answer ________

34. At steady state, the energy store in the capacitor C4 is
   A. 0 J   B. 4 J   C. 8 J
   D. 16 J   E. 18 J
   Your answer ________
Problem III. (18 points, 2 pts each)

The electrical system to the right exhibits decaying oscillations. Initially there are no currents, and the capacitors each have a voltage of 4V before SW0 is closed at t = 0. Where voltages are discussed, the sign convention is as shown by the “+” symbols. Where currents are discussed, the sign convention is from + to - through each component.

For questions 35 – 38, choose the correct answer from the choices provided.

35. The magnitude of the charge on C1 is always
   A. 0  B. half that on C2  C. equal to that on C2  D. twice that on C2  E. cannot be determined

   Your answer ________

36. The magnitude of the current through C1 is always
   A. 0  B. half that on C2  C. equal to that on C2  D. twice that on C2  E. cannot be determined

   Your answer ________

37. The magnitude of the voltage across L3 is always
   A. 0  B. half that on L4  C. equal to that on L4  D. twice that on L4  E. cannot be determined

   Your answer ________

38. The magnitude of the current through L3 is always
   A. 0  B. half that on L4  C. equal to that on L4  D. twice that on L4  E. cannot be determined

   Your answer ________

Answer Questions 39 – 43 as True or False. If a question is not NECESSARILY true, then mark it as false.

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<table>
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<tbody>
<tr>
<td>39</td>
<td>If (i_{R5} &gt; 0) then (V_{C2} &gt; 0)</td>
<td>☐ True  ☐ False</td>
</tr>
<tr>
<td>40</td>
<td>If (i_{R5} &gt; 0) then (V_{L3} &gt; 0)</td>
<td>☐ True  ☐ False</td>
</tr>
<tr>
<td>41</td>
<td>If (V_{L4} &gt; 0) then (i_{C1} &gt; 0)</td>
<td>☐ True  ☐ False</td>
</tr>
<tr>
<td>42</td>
<td>If (V_{L3} &lt; 0) and (V_{C1} &lt; 0) then (V_{R3} &gt; 0)</td>
<td>☐ True  ☐ False</td>
</tr>
<tr>
<td>43</td>
<td>If (d(i_{R5})/dt &gt; 0) then (V_{L3} &gt; 0)</td>
<td>☐ True  ☐ False</td>
</tr>
</tbody>
</table>
Problem IV. (12 pts)
Clearly write your answer in the box for each part below. No points will be awarded for indicating your answer in any other way.
Consider the system below, where the battery supplies a constant voltage \( (V_B) \). Use the current directions indicated to help write the governing equations. Before the switch is closed at \( t = 0 \), there is no current through the inductor, and the capacitor is uncharged. Refer to the voltage/current through each element by using subscripts with element names (e.g., \( V_{R1} \) and \( i_{R1} \) refer to the voltage across and current through resistor \( R_1 \), respectively).

Answer the following questions. Show your work for finding the state equations below on this page!

44. List the state variable(s) – for capacitors, use charge as the state variable. (1 pt each)

45. Write constitutive laws for each element. (4 pts)

46. Write the appropriate Kirchoff voltage law (KVL) equation(s) using the sign convention indicated in the figure. Note: all voltages should appear in at least one equation. (3 pts)

47. Write the appropriate Kirchoff current law (KCL) equations(s) using the current arrows indicated in the figure. Note: all currents should appear in at least one equation. (2 pts)
Problem V. (20 pts)
Clearly write your answer in the box for each part below. No points will be awarded for indicating your answer in any other way.
Consider the system below, where the battery supplies a constant voltage, $V_B = 6\text{V}$. Use the current directions indicated to help write the governing equations. Let $R_1 = 1\ \Omega$, $R_2 = 1\ \Omega$, $C_3 = 2\ \text{F}$, and $L_4 = 2\ \text{H}$. The switch has been opened for a very long time before it was closed at $t = 0$. Refer to the voltage/current through each element by using subscripts with element names (e.g., $V_{R1}$ and $i_{R1}$ refer to the voltage across and current through resistor $R_1$, respectively).

![Circuit Diagram]

Given the state variables are $Q_{C3}$ and $i_{L4}$, and the corresponding KCL, KVL, and CL equations as following:

<table>
<thead>
<tr>
<th>Kirchoff voltage law equations</th>
<th>Kirchoff current law equations</th>
<th>Constitutive law equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_B = V_{R1} + V_{C3}$</td>
<td>$i_{R1} = i_{R2} + i_{C3}$</td>
<td>$V_{R1} = i_{R1} R_1$</td>
</tr>
<tr>
<td>$V_{C3} = V_{R2} + V_{L4}$</td>
<td>$i_{R2} = i_{L4}$</td>
<td>$V_{R2} = i_{R2} R_2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_{C3} = Q_{C3} / C_3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_{L4} = i'_{R2} L_4$</td>
</tr>
</tbody>
</table>

48. Find the state equation(s) in terms of the state variable(s) and system parameters. (8 pts)
49. Using what you know about capacitors and inductors, what are appropriate initial conditions for the state variables when the switch is closed at time $t = 0$? (2 pts)

50. Using the forward Euler method, calculate the values of the state variables at time $t = 0.1$ s using a time step of 0.1 s. (4 pts)

51. Using what you know about capacitors and inductors, will this system reach a steady state? If so, what is the value of $i_{L4}$? What is the value of $V_{C3}$? (6 pts)
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