HW3 Solutions

1 (60 pts)
1-1 (36 pts)
(2 pts each, 4 pts total)

(2 pts each, 4 pts total) The force balance equations are

\[ m_3 a_3 = f_4 + f_5 - f_1 - f_2 - w_3, \]
\[ m_6 a_6 = F_6 - f_4 - f_5 - w_6. \]

(2 pts each, 8 pts total) The geometric continuity equations are

\[ v_6 - v_{\text{road}} = v_1 + v_4, \]
\[ v_3 - v_{\text{road}} = v_1, \]
\[ v_1 = v_2, \]
\[ v_4 = v_5. \]

(1 pt each, 4 pts total) The constitutive laws give

\[ f_1 = k_1 x_1, \quad f_2 = b_2 v_2, \]
\[ f_4 = k_4 x_4, \quad f_5 = b_5 v_5. \]

(2 pts) \( v_{\text{road}} \) is derived as

\[ v_{\text{road}}(t) = \frac{dx_{\text{road}}(t)}{dt} = 0.1 \cos(10t). \]

(1 pt each, 4 pts total) The state variables are \( x_1, x_4, v_3, v_6. \)
(2 pts each, 8 pts total) The state equations are

\[x_1' = v_1 = v_3 - v_{road},\]
\[x_4' = v_4 = v_6 - v_3,\]
\[v_3' = a_3 = \frac{f_4 + f_5 - f_1 - f_2 - w_3}{m_3} = \frac{k_4x_4 + b_5v_5 - k_1x_1 - b_2v_2}{m_3} - g\]
\[v_3' = a_3 = \frac{k_4x_4 + b_5(v_6 - v_3) - k_1x_1 - b_2(v_3 - v_{road})}{m_3} - g,\]
\[v_6' = a_6 = \frac{F_6 - f_4 - f_5 - w_6}{m_6} = \frac{F_6 - k_4x_4 - b_5v_5}{m_6} - g\]
\[v_6' = a_6 = \frac{F_6 - k_4x_4 - b_5(v_6 - v_3)}{m_6} - g.\]

(2 pts) At \(t = 0\), \(x_6 = x_{road} + x_1 + x_4\).

1-2 and 1-3 (24 pts)

(See sample codes and figs below)

(4 pts) Define system constants.
(4 pts) Define initial conditions.
(2 pts) Update \(v_{road}\).
(4 pts) Update \(v_1, v_4, a_3, a_6\).
(4 pts) Update \(x_1, x_4, v_3, v_6\).
(2 pts) Update \(x_6\) and compute difference.
(2 pts) Plot time vs \(v_6(t)\)
(2 pts) Plot time vs \(x_6(t) - x_6(0)\).
(-0.5 pt each incorrect/missing label)
clear; clear all; close all  
% system constants
k1 = 100000; % spring constant 1 (N/m)  
b2 = 10000; % damping constant 2 (N-s/m)  
m3 = 100; % mass 3 (kg)  
k4 = 20000; % spring constant 4 (N/m)  
b5 = 2000; % damping constant 5 (N-s/m)  
m6 = 60; % mass 6 (kg)  
g = 9.81; % gravity (m/s^2)  

% initial conditions
x1(1) = -0.104; % initial spring displacement 1 (m)  
v3(1) = 0; % initial velocity of mass 3 (m/s)  
x4(1) = -0.0294; % initial spring displacement 4 (m)  
v6(1) = 0; % initial velocity of mass 6 (m/s)  
t(1) = 0; % initial time 0 (s)  
dt = 0.01; % time step  
x6(1) = 0.01*sin(10*t(1)) + x1(1) + x4(1); % initial displacement 6 (m)  
xdiff6(1) = 0;  

% main loop
for i=1:200
    F6(i) = 0;  
v_road(i) = 0.1*cos(10*t(i));  
v1(i) = v3(i) - v_road(i);  
v4(i) = v6(i) - v3(i);  
a6(i) = (F6(i)-k4*x4(i)-b5*v4(i))/m6-g;  
a3(i) = (k4*x4(i)+b5*v4(i)-k1*x1(i)-b2*v1(i))/m3-g;  
x1(i+1) = x1(i) + v1(i)*dt;  
x4(i+1) = x4(i) + v4(i)*dt;  
x6(i+1) = x6(i) + v6(i)*dt;  
x1(i+1) = x1(i) + v1(i)*dt;  
x4(i+1) = x4(i) + v4(i)*dt;  
x6(i+1) = x6(i) + v6(i)*dt;  
t(i+1) = t(i) + dt;  
x6(i+1) = x6(i) + v6(i)*dt;  
xdiff6(i+1) = x6(i) - x6(1);  
end  
figure  
plot(t,v6)  
xlabel('Time (s)')  
xlim([0 2])  
ylabel('Velocity (m/s)')  
title('v6(t)')  
figure  
plot(t,xdiff6)  
xlabel('Time (s)')  
xlim([0 2])  
ylabel('Displacement (m)');  
title('x6(t)-x6(0)')
2 (20 pts)

2-1 (3 pts)
(1 pt each force, 3 pts total)

\[ \begin{align*}
\text{x: } F_{\text{friction}} - mg \sin \alpha &= 0, \\
\text{y: } N - mg \cos \alpha &= 0.
\end{align*} \]

2-2 (6 pts)
(3 pts each equation, 6 pts total) Consider the force balance along two directions, one horizontal to the ramp surface (x) and the other vertical to the ramp surface (y).

\[ \begin{align*}
\text{x: } F_{\text{friction}} - mg \sin \alpha &= 0, \\
\text{y: } N - mg \cos \alpha &= 0.
\end{align*} \]

2-3 (8 pts)
(2 pts) When the block starts to slip, it follows that

\[ F_{\text{friction}} = \mu N. \]

(2 pts each, 4 pts total) Plugging in results of 2-2, we have

\[ F_{\text{friction}} = \mu N = \mu mg \cos \alpha, \]
\[ F_{\text{friction}} = mg \sin \alpha. \]

(4 pts) Combining the two equations yields

\[ \tan \alpha = \mu = 0.4 \Rightarrow \alpha = 21.8^\circ \]

2-3 (3 pts)
(1 pt) Since 25° > 21.8°, the block will slip.
(2 pts) The acceleration is

\[ ma = mg \sin \alpha - F_{\text{friction}} = mg (\sin \alpha - \mu \cos \alpha) \]
\[ a = g(\sin \alpha - \mu \cos \alpha) = 0.59\text{m/s}^2 \]
3 (20 pts)

3-1 (12 pts)

(2 pts ts each equation, 4 pts total) The force balance equations are

\[ mg = N, \]
\[ am = f. \]

(2 pt) The kinetic energy is

\[ KE = \frac{1}{2}mv^2. \]

(2 pt) The work of friction is

\[ W = \mu mgd. \]

(2 pt) The principle of work and energy gives

\[ KE = W \Rightarrow d = \frac{mv^2}{2\mu mg} = \frac{v^2}{2\mu g} = 3.12m. \]

(An alternative way to derive the sliding distance is as follows)

(2 pt) The friction is

\[ f = \mu N. \]

(2 pts) Hence, the acceleration is

\[ a = f/m = \mu g = 7.848m/s^2. \]

(2 pts) The sliding distance is

\[ d = \frac{v_0^2}{2a} = 3.12m. \]

(2 pts) Since the player gains half of his height when sliding down, the total distance is

\[ s = d + h/2 = 4.02m. \]

3-2 (12 pts)

(2 pts) The time costed for sliding down is

\[ t_{slide} = \frac{v_0}{a} = 0.892s. \]

(2 pts) If the player keep running, the time would be

\[ t_{run} = \frac{s}{v_0} = 0.574s. \]

(2 pts) Hence, the player did not save time.

(2 pts) The time lost by sliding is

\[ t_{slide} - t_{run} = 0.318s. \]