EA3 – Week 9
Electrical Systems
RC Circuits & State Equations

Spring 2018
RQ (T/F)

1. Counting equations and unknowns
   A. is hopeless as there are so many
   B. is essential to avoid hopeless algebra
   C. helps avoid missing a loop or node equation

2. The direction of current arrows must always point in the direction of actual flow

3. A negative value associated with a current arrow means that current flows in the direction opposite the arrow

4. To find relationships for voltages, we can use
   F. loops and KVL
   G. named absolute potentials and series/parallel rules
   H. Kirchoff’s current law
Outline

• Mechanical-Electrical Analogy
• State equations part 1
  – Steps for & Finding State Equations
  – The first instant...
  – Steady State
Mechanical - Electrical Analogy

Mechanical System:
- elements in parallel: share velocity
- elements in series: share force

\textbf{Flow} : velocity \quad \textbf{Effort} : force

Electrical System:
- elements in parallel: share voltage
- elements in series: share current

\textbf{Flow} : current \quad \textbf{Effort} : voltage

For equivalence between systems, we want flows and efforts to be related in the same way.
This mechanical system is equivalent to...

(A) \[ + \text{V}_B \quad \text{Spring} \quad \text{Resistor} \]

(B) \[ + \text{V}_B \quad \text{Spring} \quad \text{Spring} \quad \text{Resistor} \]

(C) \[ + \text{V}_B \quad \text{Spring} \quad \text{Resistor} \]

(D) \[ + \text{V}_B \quad \text{Spring} \quad \text{Spring} \quad \text{Spring} \]
Mechanical - Electrical Analogy

This mechanical system is equivalent to...

(A)  

(B)  

(C)  

(D)  

(E)
(1) Draw current arrows and name the currents
Label voltmeter polarities
(2) Think about the system’s motion:

When S0 is first closed, which are zero?

(A) $i_{R1}$
(B) $i_{C2}$
(C) $i_{R3}$
(D) $V_{R1}$
(E) $V_{C2}$
(F) $V_{R3}$
(2) Think about the system’s motion:

In steady state, which are zero?

(A) $i_{R1}$  (D) $V_{R1}$
(B) $i_{C2}$  (E) $V_{C2}$
(C) $i_{R3}$  (F) $V_{R3}$
(3) What are the state variables?

(A) \( i_{R1} \)

(B) \( i_{C2} \)

(C) \( i_{R3} \)

(D) \( V_{R1} \)

(E) \( V_{C2} \) (or \( q_{C2} \))

(F) \( V_{R3} \)
So state equations look like this:

\[ V_{C2}' = \text{blah blah blah about } V_{C2} \]

(and also about Vo, R1, C2, R3)
(4) Find independent loop equations

(A) \(-V_o + V_{R1} + V_{C2} = 0\)
(B) \(+V_o + V_{R1} + V_{C2} = 0\)
(C) \(V_{R3} - V_{C2} = 0\)
(D) \(V_{C2} + V_{R3}\)
(E) \(V_o = V_{C2}\)
(F) \(V_o = V_{R1}\)
(5) Find junction equations

(A) $i_{C2} = i_{R3}$
(B) $i_{R1} + i_{R3} = 0$
(C) $i_{R1} = i_{C2} + i_{R3}$
(D) $i_{C2} + i_{R3} = 0$
(E) need to label $i_0$
(6) Write constitutive laws
(7) Collect your materials:

**Loops**

\[ +V_0 + V_{R1} + V_{C2} = 0 \]
\[ V_{R3} - V_{C2} = 0 \]

**Nodes**

\[ i_{R1} = i_{C2} + i_{R3} \]

**Constitutive laws**

\[ V_{R1} = i_{R1} R1 \]
\[ V_{R3} = i_{R3} R3 \]
\[ V_{C2} = Q_{C2} / C2 \]

and don’t forget \( Q' = i \)

Goal: \( V_{C2}' = \text{blah blah blah about } V_{C2} \)
At $t = 0$ the capacitor has a voltage of 6V across it.

a. What is the power dissipated (in both resistors together) at $t=0^+$?

b. Approximately what is the power dissipated (in both resistors together) at $t=1.33$ sec?

c. What is the energy of the capacitor at $t=0^+$?

d. Find a state equation(s) for the system. Use symbols, not values!
Outline

• State equations for a 2-state system
• Inductors
• LC Circuits
• Mechanical and Electrical Analogies
RQ (T/F)

1. Which of the following relations are true for an inductor?
   A. \( i' = L \frac{dV}{dt} \)
   B. \( V = L i' \)
   C. \( i = \frac{\int V \, dt}{L} \)

2. In steady state, an inductor wants
   D. constant voltage
   E. zero voltage
   F. constant current

3. In steady state, a capacitor wants
   G. constant voltage
   H. zero voltage
   I. zero current

4. True or False
   J. Current in an inductor can change instantly
NOW AGAIN FOR A 2-STATE SYSTEM

(1) Name the currents, label voltmeter polarities
(2) Think about system motion (at $t=0^+$ and at $t=\infty$)
(3) Identify state variables
(4) Loop equations (voltages) - KVL
(5) Node equations (currents) - KCL
(6) Constitutive laws
Why is it not worth naming $i_{??}$?

- It’s not the current going through any particular component
- That’s all one big node!
State variables: \(\{Q_{C2}, Q_{C5}\}\)

Loop equations (KVL):
\[
V_0 + V_{R1} + V_{C2} = 0 \\
-V_{C2} + V_{R3} = 0 \\
-V_{R3} + V_{R4} + V_{C5} = 0
\]

Node equations (KCL):
\[
i_{R1} = i_{C2} + i_{C5} + i_{R3} \\
i_{R4} = i_{C5}
\]
State variables: \{Q_{C2}, Q_{C5}\}

Loop & node equations:

Vo + V_{R1} + V_{C2} = 0

-V_{C2} + V_{R3} = 0

-V_{R3} + V_{R4} + V_{C5} = 0

i_{R1} = i_{C2} + i_{C5} + i_{R3}

Form of state equations:

Q_{C2}' = \text{blah blah blah about } \{Q_{C2}, Q_{C5}\}

Q_{C5}' = \text{yak yak yak about } \{Q_{C2}, Q_{C5}\}

Do this one first
Inductors

- $V_L = L\frac{di}{dt}$
- The first instant: $di = \frac{Vdt}{L}$
- Steady State: $i_L' = 0, V_L = 0$
- LC Circuits
  - Oscillatory behavior
  - Natural Frequency: $\sqrt{\frac{1}{LC}}$
- Mechanical and Electrical Analogies
When S1 is first closed, which are zero?

(A) $i_{L2}$  
(B) $i_{L5}$  
(C) $V_{L2}$  
(D) $V_{R3}$  
(E) $V_{C1}$  
(F) $V_{L5}$
In steady state, which are zero?

(A) \( i_{L2} \)
(B) \( i_{L5} \)
(C) \( V_{L2} \)
(D) \( V_{R3} \)
(E) \( V_{C1} \)
(F) \( V_{L5} \)
LRC circuits and system equivalence

These all share a current

These all share a voltage
These all share a velocity

These all share a force
is analogous to...
is analogous to...

(A)  

(B)  

(C)  

(D)  

(E)  


State variables $Q_C$ & $i_L$

KVL:

KCL:

Constitutive laws:

Diffeqs:

$Q_C' =$

$i_L' =$