EA3 – Week 3
Mechanical Systems with Masses

Spring 2018
For a mass:
A. We use Newton’s second law: Net force = ma
B. We will assume a mass to be rigid
C. Its velocity is an absolute velocity, not a relative velocity
D. Newton’s second law is not true if the mass is zero

Our rule of thumb for number of states is
D. The number of springs
E. The number of springs + masses
F. The number of springs + masses + dampers

State equations should not contain
A. velocity of a damper C. acceleration of a spring
B. damping constants D. mass values
REVIEW: The “Three step process in EA3”

Step 1
Geometric
Constitutive

Step 2
Differential Equations of Motion

Step 3
Analytic
Numerical: Euler’s method
State variables and state equations

• How many variables do we need to define the ‘state’ of a system?

   Ans: Typically **state variables** = # masses (v’ s) + # springs (x’ s)

   Caution: Do not do this blindly, seek ‘independent’ state variables.

• How many equations of motion (EOM) should we generate?

   Ans: Equal to the number of state variables.

• These are called **state equations** which are of the form:

   \[ x_s' = \text{function}(x_s, v_m) \]
   \[ v_m' = \text{function} (x_s, v_m) \ldots \]

• These are **coupled differential equations**.
Mechanical Systems

Elements
- Dampers
- Springs
- Masses

Equilibrium
Geometric Continuity
Masses (assumed rigid)

- Governed by universal law: $\Sigma F = ma$
- Governed by **absolute** velocity
- Force does not go “through”
- State variables: velocity of each mass

**Procedure for analysis:**
- Draw a connection dot at the mass
- Draw the absolute velocity of the mass (take positive to the right)
- Invoke Newton's Law for Force Balance at each mass
- Apply Continuity as usual to the other elements
Systems with masses

- Think about how the system moves
- Annotate/redraw diagram w/ connection dots, label absolute mass velocities (+ to right)
- Number all elements, write constitutive law of springs and dampers
- FBD at each connection dot, then write force balance eqns (=ma for masses, =0 otherwise)
- Write complete set of geometric continuity eqns
- List state variables: # masses (v’ s) + # springs (x’ s)
- Write eqns of motion:
  - \( v_m' = \ldots \) And \( x_s' = \ldots \)
  - May need definitions: \( v=x' \) or \( a=v' \)
  - Use FB, GC, CL’s to obtain
Which are true?

(A) \( v_1 + v_2 + v_3 + v_4 = 0 \)

(B) \( F_1 = F_3 = F_4 \)

(C) \( v_3 + v_4 = 0 \)

(D) \( m \frac{dV_2}{dt} = -F_1 + F_3 + F_4 \)

(E) \( F_1 = F_3 \)

(F) \( F_3 + F_4 = 0 \)
Which are necessarily true?

(A) if $X_1 < 0$ then $V_2 > 0$
(B) if $dV_2/dt > 0$ then $X_1 < 0$
(C) if $dV_2/dt > 0$ then $v_3 < 0$
(D) if $X_1 > 0$ then $v_3 > 0$
(E) if $V_2 = 0$ then $v_3 = 0$
(F) if $V_2 = 0$ then $X_1 = 0$
Analyzing a spring-mass-damper system

- Decide state variables
- **Step 1:** Write FBE, GCE, CL
- **Step 2:** Get differential equations for state variables

Coupled Differential Equations, e.g.
\[ x'_s = \text{function}(x_s, v_m) \]
\[ v'_m = \text{function}(x_s, v_m) \]

Identify the initial Conditions

- **Step 3:** Numerical solution: Euler method (non-matrix form)
\[ v_s(t) = x'_s(t) = \text{func1}(x_s, v_m, k, b, m) \quad \text{EOM} \]
\[ a_m(t) = v'_m(t) = \text{func2}(x_s, v_m, k, b, m) \quad \text{EOM} \]
\[ x_s(t+dt) = x_s(t) + v_s(t) \times dt \]
\[ v_m(t+dt) = v_m(t) + a_m(t) \times dt \]
Steps 1 & 2: Setting up DEQ

What are the state variables?

\[ X_1 \text{ and } V_3 \]

**Step 1:**
- **FBE:** \( F_1 = F_2 ; \ -F_2 = mV_3' \)
- **GCE:** \( V_1 + V_2 = V_3 \)
- **CL:** \( F_1 = kX_1 ; \ F_2 = bV_2 \)

**Step 2:** Find the state equations

\[ X_1' = V_3 - \frac{k}{b} X_1 \]
\[ V_3' = -\frac{k}{m} X_1 \]
Step 3: Solving DEQ by Euler’s method

State equations

\[
\begin{align*}
X_1' &= V_3 - \frac{k}{b} X_1 \\
V_3' &= -\frac{k}{m} X_1
\end{align*}
\]

Given at \( t=0.5 \)

- \( X_1(t=0.5) = 3 \)
- \( V_3(t=0.5) = 2 \)

\( X_1' \) (t=0.5) = \\
\( V_3' \) (t=0.5) = \\
\( X_1(t=0.6) = \\
\( V_3(t=0.6) = \)
The “Three step process in EA3”

Step 1
Geometric
Constitutive

Step 2
Differential Equations of Motion

Step 3
Analytic
Numerical: Euler’s method
At the moment shown (t = 0.4), Xs=-2 and Vm=3.

- Approximate $X_s(t=0.5)$
- Compute (at $t=0.4$), $F_d$ and $F_s$
- Compute (at $t=0.4$), $a_m=dV_m/dt$
- Approximate $V_m(t=0.5)$
- Is the mass speeding up or slowing down?
- Why?

**EOMs:**

$x_1' = V_2$

$V_2' = -0.5x_1 - 0.25V_2$
CQ 4

How many independent state variables does this system have and what are they?

(A) one  (B) two  (C) three  (D) four  (E) five  (F) zero
CQ 5

How many independent state variables does this system have and what are they?

(A) one
(B) two
(C) three
(D) four
(E) five
(F) zero
How many independent state variables does this system have and what are they?

(A) one  
(B) two  
(C) three  
(D) four  
(E) five  
(F) zero
CQ 7

How many independent state variables does this system have and what are they?

(A) one  
(B) two  
(C) three  
(D) four  
(E) five  
(F) zero
A system consists of a mass, experiencing a force due to gravity as shown, and supported by a spring and a damper, in parallel. All components are ideal and have linear constitutive laws.

a) How many state variables are there?
b) What are they?
c) How many state equations do you expect?
d) Write the constitutive law(s).
e) Write the GCE(s).
f) Write the FBE(s).
g) In steady state (i.e. when all state variables are constant) what are $X_2$ and $V_1$ (including sign)?
Newton’s Laws
1. Which statement(s) are correct about Newton’s laws
   A. If $\Sigma F = 0$, the particle will be stationary
   B. Particles in contact exert equal forces on each other
   C. Newton’s second law is not true if $\Sigma F = 0$.

2. Angular velocity, $\omega$,
   A. is the rate of change of angle with respect to time
   B. is always written as a vector
   C. is used to calculate “corkscrew acceleration”

3. Any object moving in a circular path experiences
   A. angular acceleration pointed toward the center
   B. normal acceleration pointed outward
   C. normal acceleration pointed toward the center
   D. no acceleration if it is moving with constant speed

4. The Bungee jumper experiences
   A. no acceleration at $t = 0+$
   B. a constant acceleration, $g$.
   C. the maximum acceleration at the bottom of her fall
5. The force of friction on an object is
   A. in the direction of slip
   B. opposite the direction of slip
   C. in the direction of the normal force
   D. opposite the direction of the normal force

6. The force of friction (Coulomb model, with $\mu > 0$) could be
   E. zero
   F. less than $\mu N$, where $N$ is the normal force
   G. equal to $\mu N$, where $N$ is the normal force
   H. greater than $\mu N$, where $N$ is the normal force

7. Coulomb friction is
   I. a “Great Truth”
   J. a useful approximation
Outline

• Newton’s Laws

• Velocities and accelerations
  – Tangential and normal components
  – Normal acceleration with motion on curved path

• Center of Mass: $\Sigma F_{\text{ext}} = m_{\text{tot}} \cdot a_{\text{CM}}$

• Friction
A pellet of mass $m$ is blown through a frictionless curved pipe of radius $R$ and is expelled from the open end at the top. While in the pipe, the speed of the pellet is a constant $V$.

When the pellet is expelled, trajectory is:

(A) Curves up
(B) straight
(C) Curves down
(D) cannot be determined
A pellet of mass $m$ is blown through a frictionless curved pipe of radius $R$ and is expelled from the open end at the top. While in the pipe, the speed of the pellet is a constant $V$.

At point $P$, $v_x$ of the pellet is:
(A) Positive
(B) zero
(C) negative
(D) cannot be determined
A pellet of mass $m$ is blown through a frictionless curved pipe of radius $R$ and is expelled from the open end at the top. While in the pipe, the speed of the pellet is a constant $V$.

At point $P$, acceleration of the pellet is:

(A) Up (+y)
(B) Down (-y)
(C) Right (+x)
(D) Left (-x)
(E) Combination of above
(F) zero
A fixed (not moving) coordinate system $XYZ$ is shown. At the moment illustrated the ball is positioned at the origin of the coordinate system, with $X$ radial, $Y$ circumferential, and $Z$ vertical.

The ball moves with constant angular velocity, $\omega$. The following terms pertain to the kinematics of the ball. $a$ is its acceleration, $v$ is its velocity, and $\alpha$ is the angular acceleration of the ball.

Which quantities are nonzero at this particular moment?

(A) $a_x$  (B) $a_y$  (C) $a_z$  (D) $v_x$  (E) $v_y$  (F) $\alpha$
The heavy ball is held stationary by two taut ropes. Someone cuts rope A.

Immediately after rope A is cut, the tension in rope B is:

(A) zero
(B) mgsin45°
(C) more than mg sin 45°, because of centripetal acceleration
(D) mg
(E) insufficient information to solve
A 3kg bird is initially stationary on a plank. A cat frightens the bird, who then runs back and forth frantically on the 30 kg plank, 2 meters long. The plank is free to roll (without friction) on wheels. Whenever the bird runs left, the plank moves right, and vice versa. If the bird runs repeatedly from end to end of the plank, approximately how far does the CM of the whole system (bird + plank) wiggle back and forth?

(A) zero  (B) 0.2 meters  (C) 1 meters  (D) 2 meters
Friction!

- Draw relevant FBD
- Determine if system moves
  - Find $f_{\text{equil}}$ required for system to remain stationary
  - Compare $f_{\text{equil}}$ with $f_{\text{max}} = \mu_s * N$
- If stationary: $f = f_{\text{equil}}$ -> (No slip)
- If in motion: $f = \mu_k * N$ -> (Slip)
- Solve problem
A truck starts from “rest” on a yellow barge and drives slowly to the left. What force got the truck moving to the left?
(A) It’s not due to force, it’s due to torque from the motor
(B) The force of friction acting to the left
(C) The force of friction acting to the right.

How big is the force of friction acting on the tires?
(D) Zero, assuming the wheels roll and don’t slip
(E) Less than $\mu mg$, assuming the truck isn’t “burning rubber”
(F) $\sim \mu mg$
A 10-ton/18-wheel truck and a 1-ton/4-wheel car are going 60 mph. Both slam on the brakes at the same moment. Both vehicles’ brakes lock all wheels, and the rubber-asphalt coefficient of friction is 0.42.

(A) The car stops in a shorter distance
(B) The truck stops in a shorter distance
(C) The stopping distance is the same
(D) It depends on the height of the CM above the road surface
(E) It depends which one has been traveling longer at 60 mph
What is the minimum force $F$ required to cause any motion?

(A) less than 9 lbs  (B) 9 lbs  (C) more than 9 lbs

Once motion begins, the force of friction acting on the upper (yellow) block is

(D) less than 7 lbs  (E) 7 lbs  (F) more than 7 lbs
As the angle $\theta$ is increased slowly from 0 to 89 degrees, when does sliding happen?

(A) the cube will begin to slide at $\theta < 45^\circ$
(B) the cube will begin to slide at $\theta = 45^\circ$
(C) the cube will begin to slide at $\theta > 45^\circ$
(D) the cube will never slide below 90 degrees
(E) there is no way to tell
As the angle $\theta$ is increased slowly from 0 to 89 degrees, when does tumbling happen?

(A) the cube will begin to tumble at $\theta < 45^\circ$
(B) the cube will begin to tumble at $\theta = 45^\circ$
(C) the cube will begin to tumble at $\theta > 45^\circ$
(D) the cube will never tumble below 90 degrees
(E) there is no way to tell
Work It Out! (2006, Quiz 1)

The lemur on a unicycle is riding backwards down the slope, opposite the arrow shown in the picture. (Choose A, B, C, or D for each blank.)

- Which is true regarding the force of friction acting on the wheel? ____
- If the lemur rides up the hill at a constant speed, then which is true of the force of friction on the wheel? ____
- If the lemur is slowing down while going up the hill, which is true of the friction force acting on the wheel? ____

A) The direction of friction is up the hill.
B) The direction of friction is down the hill.
C) The force of friction is zero.
D) The force of friction cannot be determined from the provided information.
Quiz 1 is next week
- Topics: All lecture material through and including this week (see webtext).
- See “Quizzes & Review” link for extra practice
- NOTE: no calculators, no phones, no notes, no cheat sheets

Quiz proctoring - wandering eyes, talking, phones out, rummaging through backpacks, etc.
- 1\textsuperscript{st} offense: warning + name taken down
- 2\textsuperscript{nd} offense: loss of 50\% + referred to Academic Integrity office