1. (70 points) The system shown consists of a damper, a spring, a lever, and a massive block.
The lever has total length 30 cm, with a fulcrum at one end. The attachment points are 10 cm and 30 cm from the fulcrum (20 cm from one attachment point to the other).

v3 and v4 refer to the velocities of the two attachment points as shown. The forces experienced by the lever at those attachment points are f3 and f4 as shown. Please use the positive definitional arrows and variable names shown.

You are given that the constitutive laws of the spring and damper are f1 = b1 v1 and f2 = k2 x2

1-1) Write equation(s) relating v3, v4, f3 and f4, due to the lever.

1-2) Write the force balance and geometric continuity equation(s) (do not include equations you already wrote above).

1-3) Find the state equation(s).

1-4) Reduce the state equation(s) to a single differential equation for the spring displacement. Solve this equation analytically and obtain a solution for the following initial conditions: x2(t=0) = 1 and x2′(t=0) = 1; and parameters: m5 = 1/9, k2 = 1, b1 = 1.

2. (30 pts) John wants to move a 200-kg rock using a stiff bar measuring 3 meters long as a lever. He puts the fulcrum x meters from the rock. Assume that g = 10m/s^2.

2-1. Sketch this in a diagram and annotate the forces/velocities. (10pts)

2-2. If x = 0.5m, how much force must he use in order to move the rock? (10pts)

2-3. If the maximum force he can exert on the crowbar is 500N, what is the maximum value of x such that he can still move the rock? (10pts)