HW 2 SOLN

1-1 (Step 0 \rightarrow \frac{1}{2} \rho t \text{ each})

For A

\[ \begin{align*}
   f_1 &= k_1 x_1 \\
   f_2 &= k_2 x_2 \\
   f_3 &= b_3 v_3 \\
   f_4 &= b_4 v_4 \\
\end{align*} \]

For B

\[ \begin{align*}
   f_1 &= f_2 + f_3 \\
   f_2 + f_3 &= f_4 \\
\end{align*} \]

(Step 1 \rightarrow \frac{1}{2} \rho t \text{ each})

For A

\[ \begin{align*}
   V_2 &= V_3 \\
   V_1 + V_2 + V_4 &= 0 \\
   f_2 + f_3 &= f_4 \\
\end{align*} \]

For B

\[ \begin{align*}
   f_1 &= f_2 + f_3 \\
   f_2 + f_3 &= f_4 \\
   m_5 a_5 &= f_4 \\
\end{align*} \]

(Step 2 \rightarrow 1 \rho t \text{ per state vbls})

\[ \begin{align*}
   \text{state vbls:} & \quad x_1, x_2, V_5 \\
\end{align*} \]

For A

\[ \begin{align*}
   x_1' &= V_1 = V_2 - V_4 \\
   x_2' &= V_2 = V_3 \\
   x_3' &= V_1 = -V_3 - \frac{f_4}{b_4} \\
   &= -\frac{f_3}{b_2} - \frac{f_1}{b_4} \\
   &= -\frac{(f_1-f_2)}{b_3} - \frac{f_1}{b_4} \\
\end{align*} \]

For B

\[ \begin{align*}
   x_1' &= V_1 = V_2 - V_4 \\
   x_2' &= V_2 = V_3 \\
   x_3' &= V_5 = \frac{k_1 x_1 - k_2 x_2}{b_3} \\
   &= \frac{k_1 x_1}{b_3} - \frac{k_2 x_2}{b_3} \\
   V_5 &= \frac{k_1 x_1}{b_3} \\
   V_5 &= \frac{-k_1 x_1}{b_3} \\
   V_5 &= \frac{-k_1}{m_5} x_1 \\
\end{align*} \]

Please note that the solutions are prepared by TAs - feel free to come to EA3 office hours if you have questions.
HW 2 SOLN

1-2: (1/2 pt for element) (1/2 pt for reason)
not needed - rigid mass is anchored to rigid wall

C

not needed - damper in parallel with rigid mass means damper can't lengthen or shorten

D

not needed - damper not connected on RHS, so no velocity of v4

1-3: (1 pt each) -> ok if elements are numbered differently

C

D

E

See next page for associated FBD's, GC's, FB's, CL's

2

(a) T
(b) T
(c) T
(d) T

(k) T \rightarrow v_5 = v_2
(l) T \rightarrow x_4 > 0 \rightarrow f_5 > 0 \rightarrow v_7 > 0 \quad (v_2 + v_6 + v_7 = 0 \text{ from GC's})
(e) T

(m) F \rightarrow m_2 \text{ can move right but accelerate left}
(f) F

(n) T \rightarrow a_4 < 0 \rightarrow f_3 > 0 \rightarrow x_3 > 0
(g) F

(o) F \rightarrow not enough info
HW 2 SOLN

For use with answering Prob #2

\[ \begin{align*}
G C's & : & V_1 = V_5 = V_2 \\
& & V_3 = V_4 - V_2 \\
& & V_5 + V_6 + V_7 = 0
\end{align*} \]

\[ \begin{align*}
V_1 = V_6 = -V_2 \\
V_3 = V_4 - V_2 \\
V_5 + V_6 + V_7 = 0
\end{align*} \]

\[ \begin{align*}
F B & : & m_2 a_2 = -f_1 - f_5 + f_3 + f_6 \\
& & f_6 = f_7 \\
& & m_3 a_3 = -f_3
\end{align*} \]

\[ \begin{align*}
f_1 &= k_1 x_1 \\
f_3 &= k_3 x_3 \\
f_5 &= b_5 v_5 \\
f_6 &= k_6 x_6 \\
f_7 &= b_7 v_7
\end{align*} \]

\[ \text{DEQ: } 3 x' - 18 x = 0 \rightarrow x' - 6 x = 0 \rightarrow x' = 6 x
\]

Try \( x = A e^{bt} \) \{ plug into DEQ \:

\[ x' = A B e^{bt} \]

\[ x = A B e^{bt} \]

Thus, (a) NO 
(b) NO 
(c) YES 
(d) NO 
(e) YES 
(f) NO

1 pt each correct answer.

3-2 To find particular soln, plug IC into general soln

\[ \begin{align*}
IC: x(0) & = 10 = A e^{b(0)} \\
10 & = A (1.822) \\
A & = 5.49
\end{align*} \]

Thus, \( x(t) = 5.49 e^{6t} \) or \( x(t) = 5.5 e^{6t} \)
HW 2 SOLN

4-1  Part I
Step 1 (1 pt each)
\[ \begin{align*}
\text{FB} & \quad f_1 - f_2 = 0 \\
\text{GC} & \quad V_1 = V_2 \\
\text{CL} & \quad f_1 = k_1 x_1, \\
& \quad f_2 = b_v V_2 \\
\end{align*} \]

4-2  Step 2 (4 pts)
\[ x'_1 = V_1 = V_2 = \frac{f_2}{b_2} = -\frac{f_1}{b_2} = -\frac{k_1 x_1}{b_2} \Rightarrow \boxed{x'_1 = -\frac{k_1}{b_2} x_1} \]

4-3  Step 3 (6 pts)
(See Matlab code + plots for numerical solution)

4-4  Step 3 (Analytic) (8 pts \rightarrow 1 pt partial credit for each underlined part)
Try \( x_1 = Ae^{bt} \) plug into DEQ: \( A b e^{bt} = \frac{-k_1}{b_2} Ae^{bt} \Rightarrow b = \frac{-k_1}{b_2} = -1.5 \text{ s}^{-1} \)

General soln: \( x_1(t) = Ae^{-1.5t} \)

IC: \( x_1(0) = 0.03 \)

\[ \begin{align*}
x_1(0) = 0.03 &= Ae^0 \\
A &= 0.03 \\
x_1(t) &= 0.03 e^{-1.5t} \\
\end{align*} \]

Discussion: In the numerical solution, when \( dt \) is smaller, it is closer to the exact (analytic) solution. This is because the error in the Euler Method solution decreases as \( dt \) is reduced (i.e., assumption of \( V_1 = \text{const} \) for small time steps is a better approximation for smaller \( dt \)).
clear;
k = 1500;
b = 1000;

%Time step size 1 (dt = 0.5)
dt1 = 0.5;
t1(1) = 0;
x1(1) = 0.03;
for i = 1:10
    v1(i) = (-k/b)*x1(i);
    x1(i+1) = x1(i) + v1(i)*dt1;
    t1(i+1) = t1(i) + dt1;
end

%Time step size 2 (dt = 0.1)
dt2 = 0.1;
t2(1) = 0;
x2(1) = 0.03;
for i = 1:50
    v2(i) = (-k/b)*x2(i);
    x2(i+1) = x2(i) + v2(i)*dt2;
    t2(i+1) = t2(i) + dt2;
end

texact = [0:0.01:5];
xexact = 0.03*exp(-1.5*texact);
plot(t1, x1, '-.b', t2, x2, '--r', texact, xexact, '-k')
xlabel('time (s)')
ylabel('Spring displacement (m)')
legend('dt = 0.5s','dt = 0.1s', 'Analytic Solution')
Part 2

4.5 Step 1: (1 pt each)

\[ m_3 a_3 = -f_1 - f_2 \]

FB

G: C

CL

\[ f_1 = k_1 x_1 \]
\[ f_2 = b_2 v_2 \]

4.6 Step 2: state var's: \( x_1, v_3 \)

(2 pts each EOM)

\[ x_1' = v_1 \Rightarrow x_1' = V_3 \]

\[ V_3' = \frac{1}{m_3} (-f_1 - f_2) = \frac{1}{m_3} (-k_1 x_1 - b_2 v_2) \]

4.7 Step 3: Numerical Sol'n \rightarrow See Matlab code & plots

Discussion:

First the mass speeds up because the compressed spring pushes it to the right (+V direction). At later times (as the spring approaches its equilibrium), the damper (which is in tension as the mass moves) slows the mass down.
clear;
k = 2000;
b = 1200;
m = 100;

dt = 0.01;
t(1) = 0;
xs(1) = -0.02;
Vm(1) = 0;

for i = 1:300
    vs(i) = Vm(i);  %from spring EOM (xs')
    am(i) = -(1/m)*(k*xs(i) + b*Vm(i));   %from mass EOM (Vm')
    xs(i+1) = xs(i) + vs(i)*dt;   %Euler method for spring
    Vm(i+1) = Vm(i) + am(i)*dt;   %Euler method for mass
    t(i+1) = t(i) + dt;
end

subplot(2,1,1)
plot(t, xs)
xlabel('time (s)')
ylabel('Spring displacement (m)')

subplot(2,1,2)
plot(t, Vm)
xlabel('time (s)')
ylabel('Mass velocity (m/s)')