2-1. (10 pts)

\[ f_{\text{friction}} = \mu_k N = 0.8 \times M \times 9.81 \]

Work done by friction: \( f_{\text{friction}} \times D_{\text{sliding}} \)

Initial KE: \( \frac{1}{2} MV^2 = f_{\text{friction}} \times D_{\text{sliding}} \) (4 pts)

\[ D_{\text{sliding}} = \frac{\frac{1}{2} MV^2}{\mu_k Mg} = \frac{\frac{1}{2} \times (7)^2}{0.8 \times 9.81} = 3.12 \text{ (m)} \] (4 pts)

Distance from base: \( D_{\text{from base}} = D_{\text{sliding}} + \frac{1}{2} \text{body length} = 3.12 + 0.9 \)

\[ = 4.02 \text{ (m)} \] (2 pts)

2-2. (10 pts)

No (2 pts)

\[ t_{\text{sliding}} = \frac{MV}{\mu_k Mg} = \frac{7}{0.8 \times 9.81} \approx 0.8925 \text{ (3 pts)} \]

\[ t_{\text{running}} = \frac{D_{\text{from base}}}{V} = \frac{4.02}{7} \approx 0.5745 \text{ (3 pts)} \]

Running quicker

Time lost by sliding: \( 0.892 - 0.574 \)

\[ \approx 0.318 \text{ (s)} \] (2 pts)
m_1 = 1500 \text{ kg} \quad m_2 = 2500 \text{ kg}

m_1 \alpha = m_1 \text{mg} \quad \Rightarrow \quad \alpha = \frac{m_1 \text{g}}{m_1} = 8 \text{ m/s}^2

v = \sqrt{2as} = \sqrt{2 \times 8 \times 15} = 4 \sqrt{15} \text{ m/s}.

x \text{ direction:} \quad v_x = v \cos 33 = v \times 0.6 = \frac{12}{5} \sqrt{15} \text{ m/s}.

v_y = v \sin 33 = v \times 0.8 = \frac{16}{5} \sqrt{15} \text{ m/s}.

\text{Ax mom: } \quad m_1 v_1 = (m_1 + m_2) v_x \rightarrow \text{ incorrect}

v_1 = \frac{m_1 + m_2}{m_1} v_x = \frac{5}{3} \cdot \frac{12}{5} \sqrt{15} = \frac{32}{5} \sqrt{15} = 24.78 \text{ m/s}.

\text{Ay mom: } \quad m_2 v_2 = (m_1 + m_2) v_y

v_2 = \frac{m_1 + m_2}{m_2} v_y = \frac{8}{5} \cdot \frac{16}{5} \sqrt{15} = \frac{128}{25} \sqrt{15} = 19.83 \text{ m/s}.

\bar{v}_1 = 24.78 \text{ m/s} = 89.21 \text{ km/h} > 60 \text{ km/h.} \quad \text{(limit)}

\bar{v}_2 = 19.83 \text{ m/s} = 71.39 \text{ km/h}

\text{if any of the calculated results does not have the right unit, -0.5 for each.}
4.3. Sol:

4-1. (a) F because it is a perfectly elastic collision (2 pts)
     (b) T
     (c) F

4-2. \[ m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \] (x) (1 pt)

\[ 2 \times 2 + 1 \times 1 = 2 \times v'_A + 1 \times v'_B \]
\[ \Rightarrow 2v'_A + v'_B = 5 \] (1)

\[ \text{elastic: } \frac{v'_B - v'_A}{v_A - v_B} = 1 \Rightarrow v'_B - v'_A = 1 \] (2) (1 pt)

Combining (1) and (2) gives

\[ v'_A = \frac{4}{3} \text{ m/s} \quad v'_B = \frac{7}{3} \text{ m/s} \] (1 pt)

4-3. \underline{plastic: } v'_B = v'_A, momentum eqn. same as abae (x)

\[ 4 + 1 = 2 v'_A + v'_A \]
\[ \Rightarrow v'_A = v'_B = \frac{5}{3} \text{ m/s} \] (2 pts)

4-4. They won't separate because they have the same coefficient of friction. (2 pts)

\[ f = \mu N = \mu mg \]
\[ -f = ma \]
\[ \Rightarrow a = -\mu g = -0.5 \times 9.81 = -4.9 \text{ m/s}^2 \] (3 pts)

\[ N = mg \]

\[ a = \frac{-mg}{m} = -g = -4.9 \text{ m/s}^2 \]
\[ v_{\text{final}} = v_0 + at = 0 \quad (2\text{pts}) \]

\[ \Rightarrow t = -\frac{v_0}{a} = -\frac{5.3}{-4.9} \]

\[ = 1.09 \text{ s.} \quad (1\text{ pt}) \]
Problem 5: 4.
Credit breakdown:
Code (X Npt). Partial credit is given for incomplete code (give as in % completed)

5-1 (5pt)
3pt for EOM, 2pt for power expression (either form is acceptable)

5-2 (15pt)
Zero force profile (F(t) = 0):
- 1pt for each plot of Xs, Vs, Energy in spring, Power in spring, and Power in damper
- 2pt for discussion

Non-zero force profile (F(t) as in plot):
- 1pt for each plot of Xs, Vs, Energy in spring, Power in spring, and Power in damper
- 3pt for discussion

5-3 (5pt)
EOM with mass (5pt). Partial credit is given if intermediate steps (CL, GC, FB, FB graph) are clearly shown.
5-4 (15pt)
- 2pt for each plot of Xs, Vs, Energy in spring, Power in spring, and Power in damper
- 5pt for discussion

5-5 (5pt)
- 2pt for a NO
- 3pt for reasons

5-1:

Equation of motion:

CL: Fs = k*Xs, Fd = b*Vd
GC: Vs = Vd
FB: Fs + Fd = F(t)

Xs' = Vs = Vd = Fd/b = (F(t) - Fs)/b

\[ x_s' = \frac{F(t) - k \cdot x_s}{b} \]

Power:
Ps = Fs*Vs = k*Xs*Vs
Pd = Fd*Vd = (F(t) - Fs)*Vd = (F(t) - k*Xs)*Vs

OR

\[ P = \frac{du}{dt} = \frac{\Delta u}{\Delta t} = \frac{U_{i+1} - U_i}{\Delta t} \]

where U is the kinetic energy in sprint:
U = 0.5*k*Xs^2
Figure 1 $F(t) = 0$. The results for spring displacement match the analytical solution (the two lines overlap in the spring-displacement plot). The power flowing into the spring is negative (flowing out of spring), while power flowing into the damper is positive. These two values are equal and opposite: power flows out of the spring and into the damper at the same rate during motion. Also, note that the power flowing into the spring can be calculated either as $F_\text{v}$ or as $\Delta U/\Delta t$, where $U$ is the energy stored in the spring.
Figure 2 Non-zero F(t). The spring starts compressed by \(-0.02\), then expands at approximately constant velocity until \(t=2\) sec. At this point, the velocity falls rapidly as the force is reduced, becoming negative at \(~2.5\) sec. At \(t=4\), the force has gone back to zero and the velocity and displacement decay exponentially to zero as expected.

Figure 3 The energy in the spring, which is related to the square of its displacement, starts slightly positive due to the small initial negative displacement. The energy then dips down to zero as the spring passes through its equilibrium length and extends. As the spring extends, the energy builds to a maximum coinciding with the maximum displacement. The spring power, which is the time derivative of the energy, increases linearly until \(2\) sec. It then becomes negative with the spring velocity before decaying back up to zero when the force becomes zero. The spring and damper power are no longer equal in magnitude as was the case for \(F=0\). Note that the damper power is always \(>0\) because the damper cannot store energy, only dissipate it.
5-3 Equation of motion for case with mass, \( m \).

CL:
\[
f_s = kx_s
\]
\[
f_d = bv_d
\]

GCE: \( v_s = v_d = v_m \)

FB:
\[
m \dot{v}_m = F(t) - f_s - f_d
\]
\[
m \ddot{v}_m = \frac{F(t) - kx_s - bv_d}{m}
\]

\[
\dot{v}_m = \frac{F(t) - kx_s - bv_d}{m}
\]

5-4

Figure 4 With the addition of the mass, the velocity quickly reaches an approximately constant value, while the displacement increases approximately linearly. At 2 sec, the force begins to decrease. At this point the slope of the spring displacement begins to decrease, eventually becoming negative. At 4 sec, the force has fallen back to zero and the spring displacement and velocity decay exponentially to zero.
Figure 5 As before, the energy in the spring initially decreases as it passes from negative displacement through its equilibrium length. It then builds up to a maximum corresponding to the maximum displacement before falling. After an initial decrease, the spring power builds approximately linearly with the force until 2 sec. At this point the spring and damper power begin to fall. While the spring power becomes negative, the damper power always remains positive. This is because the damper cannot store energy, only dissipate it.

5-5
The code used for the case with the mass does not work for the case with zero (or minimal) mass. This is because the equation of motion for the acceleration of the mass involves a $1/m$ term (see part 5-3). Thus the acceleration blows up for small mass, $m$.

```matlab
% Alex Birdwell
% EA3 Homework 4 Problem 6
% April 26, 2006

clear
% close all
clc

k = 2000; % N/m
b = 1200; % N-s/m
m = 100; % kg

xs(1) = -0.02; % initial spring compression, meters
vm(1) = 0;
t(1) = 0; % initial time is zero
dt = 0.01; % timestep of 0.01 sec

Ft(1) = 0; % initial applied force
energy_s(1) = 0.5 * k * xs(1)^2;
xexact(1) = -0.02;
```
for i = 1:(8/dt)  % want to run for 8 seconds
    t(i+1) = t(i) + dt;  % next time

    if t(i+1) <= 2
        Ft(i+1) = 75 * t(i+1);
        % Ft(i+1) = 0;
    elseif t(i+1) <= 4
        Ft(i+1) = 300 - 75*t(i+1);
        % Ft(i+1) = 0;
    else
        Ft(i+1) = 0;
    end

    a(i) = (Ft(i) - k*xs(i) - b*vm(i))/m;
    vm(i+1) = vm(i) + a(i)*dt;
    xs(i+1) = xs(i) + vm(i)*dt;
    vs(i) = vm(i);
    % vs(i) = (Ft(i) - k*xs(i)) / b;  % vel of spring, m/sec
    Ps(i) = k * xs(i) * vs(i);  % power of spring, J/sec
    Pd(i) = b * vs(i)^2;  % power of damper, J/sec
    % xs(i+1) = xs(i) + vs(i)*dt;  % next position
    energy_s(i+1) = 0.5 * k * xs(i+1)^2;
    power_s_dt(i) = (energy_s(i+1) - energy_s(i))/dt;  % power in
    % spring from dU/dt
    % xexact(i+1) = -0.02 * exp(-k/b.*t(i+1));

end

figure(7);
% subplot(3,1,1); plot(t(1:end-1),xs(1:end-1),{'g--',t(1:end-1),xexact(1:end-1),{'k-.'}})
subplot(3,1,1); plot(t(1:end-1),xs(1:end-1))
xlabel('time, sec'); ylabel('X_s_p_r_i_n_g, m');
eval(['title(''title'',' almost num2str(dt) ' sec')'])
% legend('dt = ' num2str(dt) ' sec')
% legend('dt = 0.01', 'exact', 'Location', 'SouthEast')

subplot(3,1,2); plot(t(1:end-1),vs);
xlabel('time, sec'); ylabel('V_s_p_r_i_n_g, m');

subplot(3,1,3); plot(t(1:end-1),Ft(1:end-1));
xlabel('time, sec'); ylabel('Applied Force, N');

figure(8);
subplot(3,1,1); plot(t(1:end-1),energy_s(1:end-1))
xlabel('time, sec'); ylabel('Spring Energy, J')

subplot(3,1,2); plot(t(1:end-1),Ps,'r--',t(1:end-1),power_s_dt,'b-.')
xlabel('time, sec'); ylabel('Spring Power, W')

subplot(3,1,3); plot(t(1:end-1),Pd)
xlabel('time, sec'); ylabel('Damper Power, W')